What is percolation

'Percolation' as name suggests deal with process of ~~stone~~ liquid moving through cracks and pores of porous material.
So mathematically; it deals with connected clusters of randomly generated graphs.

For ex. let there is a Z^2 lattice.

This can have $4 \times 5 \times 2 = 40$ total bonds
Let; we, ~~bar~~ with probability p place one bond and $(1-p)$ we keep neighbouring donts unconnected.
And we allow an ant. to travel only across connected bonds

OR another way to look at it is: we have fully connected Z_2 lattice and we choose bonds randomly with probability p. and make a random network.
And let ant do random walk in that ~~ele~~ network starting from arbitrary origin. Question is; what will be time dependence of $\langle R^2 \rangle \leftarrow$ RMS distance.
OR; How far ant will go (ofc. RMS) after N steps.

<u>Idea</u> for $p \ll 1$ we have very few connected links.
Hence ant cannot go much further and will be restricted to that cluster (~~a~~ 2 or more connected bonds)
But as p_0 increases; cluster size & no. increases and we get more $\langle R^2 \rangle$, at $p=1$ (full lattice) we perform Random walk on Z^2 and recover back known Results: Ant 'diffuses' on Z^2.

Question is; is there an probability $p_c' < 1$ for which ant can be diffused?

Another ex.

Forest fire.

Let: we form Z_2.

and on each site; with prob. p is ~~black~~ we plant a tree and $(1-p)$ we don't.

Thus giving rise to forest.

Let; This nice forest has a tree that catches ~~forest~~ fire; thus;

for such fire to spread and become forest fire, ~~it~~ it shall have another tree neighbouring; which can ~~will~~ spread fire to neighbouring trees and so on;

"Show different p"

Now; we check for 'lifetime' of forest fire.

~~we do Monte-Carlo.~~ → At each T: fire spreads to next neighbour

Let; 0 → No tree;

1 → Tree

$\tilde{1}$ → ignited

2 → Burning

X → Black.

0	1	1	0	0
0	1	0	1	0
0	1	1	1	0
0	0	0	1	1
0	0	1	0	0

T=0; first row ignited

T=1; first row burning which ignites (2,2)

T=2; (2,2) burning, ignites (3,2)

T=3; (3,2) —— " —— (3,3)

T=4; (3,3) —— " —— (3,4)

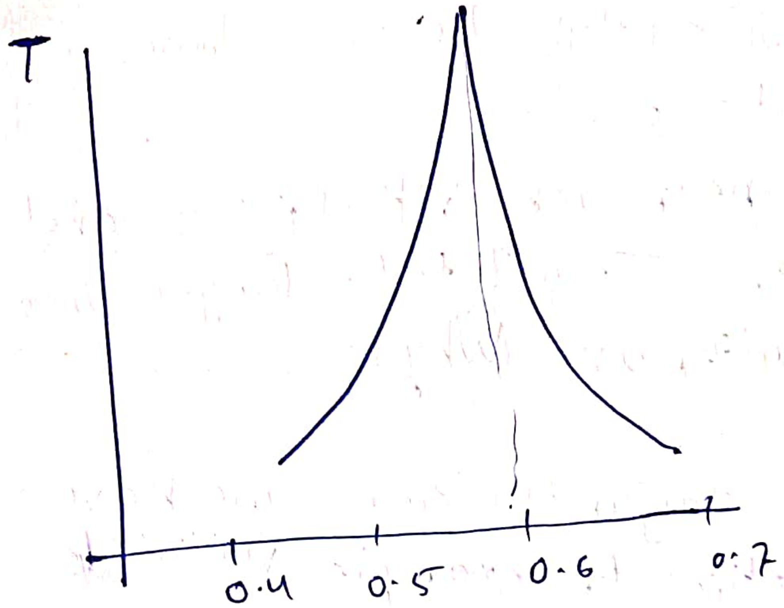
T=5; (3,4) —— " —— (2,4) & (4,4)

T=6; (2,4) & (4,4) —— " —— (4,5)

T=7; (4,5) Burnt:

⟶ 7 is Lifetime.

Then If we average over such T for given p for very large forest; we see graphs like.



Why does it peak near some finite p? What is exactly happening there? "We call this point $p_c \rightarrow$ critical point"

Check intuitively why shall T be small for Large and small p.

We can modify problem to Next nearest (decreases p_c) OR; two neighbouring burnt trees can ignite tree (ignites p_c)

* Just $p < p_c$; there are very long paths; but not quite long enough so; fire backtracs and alot of cases it has to consider or spanning/percolating

Just $p > p_c$; We have an infinite path which makes $T \propto$ Length of that path. Thus reducing timescale even though path might not be straight; it aids fire to spread not stop in b/w.

Just to state $p = p_c$ is state of 'Phase Transition'. Behaviour changes qualitatively for some parameter p.

This divergence is related to 'critical slowing down'.

Critical Slowing Down:

Responses of system becomes slower as it approaches critical point.

If we are near such critical points and we vary some parameter; It will take longer time to reach an equilibrium. Why??

at critical point; say in this case; we have a local perturbation; for example ~~varied p~~ fire spreading; for $p<p_c$; fire spreadind doesn't have much direction & paths ~~&~~ & for $p>p_c$; our lattice is almost fully connected; thus fire spreaded will just span whole network / forest nicely. But; near $p=p_c$; it has (by has it may / ~~may not~~ have) chance to go through a crooked path which would be very long to spread fire; thus slowing down the rate at which fire is being spreaded since; we have very long correlation b/w chain of trees

Relation of relaxation time (τ):

$$\tau \propto |x-x_c|^{-\nu z}$$

ν, z are critical exponents

$z \rightarrow$ dynamic

$\nu \rightarrow$ correlation

Why CSD? Since near CSD: response fn becoms slower; we can recognize whether a system is about to reach critical point or not by studying its relaxation time. which is helpful in many contexts.

Ex. Ecology.

Amazon: phase 1: forest.
-"- 2: Savannah

Disturbance: Drought, Forestfire, deforestation.
Recovery: Return to original values
Conclusion: Amazon Degrading.

Ex. Critical Opalescence..

Liquid- Gas.

at T_c; ξ diverges; fluctuations at all length scales
Fluctuation: regions where fluid is denser/rarer.
Since these fluctuations ~ light; scattering & milky
Time Delay: Since entire volume is spanned;
Time needed for these fluctuations to decay become extremely long:
S.K takes hours/days to recover.

Clusters

With enough motivation set up for why existence of infinite such path is important; we head into mathematics of it

Also; we have seen both site (Forest fire) and Bond (Ant) percolation:

We define cluster (site): when 2 or more black points are neighbouring & (bond): when from given lattice point we can access points other than nearest neighbours through connected link.

~~In~~ Infinite cluster is indicator of system in consideration being percolating or not.

Hence; we ~~co~~ define 2 things:

1. $\theta(p)$ probability of existence of such an infinite cluster.
2. Phases (Later we will see why this description is valid)

   phase 1: $p < p_c$: subcritical

   phase 2: $p > p_c$: supercritical

   We will look at what they indicate later.

$$\boxed{p_c := \sup\{p : \theta(p) = 0\}} \quad \boxed{\theta(p) \begin{cases} = 0 & \text{if } p < p_c \\ > 0 & \text{if } p > p_c \end{cases}}$$

Now we ask what is value of $p = p_c$ where $\theta(p) \neq 0$

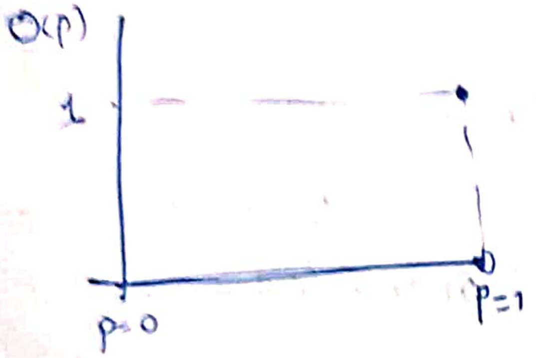
Proves and Arguments in next sections:

1. $\theta(p) \neq 0$: p_c? 2. How many ∞-clusters $\longrightarrow 1$
3. How fast phase transition occur $\longrightarrow$ on $\mathbb{Z}^2$: $\propto n^{-3/4}$
4. How large are cluster before they merge $\propto (p - p)^{-43/18}$

Let us look at 1D intuitively first.

∞-cluster.

even if one link is absent, it is not a percolating cluster. More importantly we are in Thermodynamic limit $N \to \infty$; so; even if $p<1$ by small $\varepsilon > 0$; No. of ~~open~~ closed bonds $= N\varepsilon \to \infty$ so; only for $p=1$; we get such cluster Hence No phase transition.



But for 2D (Z^2 lattice) will $\theta(p)$ be 1? (for $p_c = 1$) or it will be less than 1; If yes; how $\theta(p)$ varies with p.

Note: $\theta(p)$ denotes order as absence of ∞ cluster causes small, many clusters making system disorder but ∞-cluster introduces order.

Pierels' Argument

Notation: $P_p[A]$: prob. that event A occurs in Bernoulli percolation with parameter p.

ex. P_p[infinite cluster] ~~$= \theta(p)$~~

Thm: There exists $p_c \in (0,1)$ s.t. for $p < p_c$: $P_p[\infty] = 0$ ~~$\theta(p) = 0$~~

$p > p_c$: $P_p[\infty] = 1$ ~~$\theta(p)$ to~~ $= 1$

Pierels argument provides Bound.

- Fix origin (0)
- Define event: cluster to which '0' belongs is infinite

But $\mathbb{P}_p$[cluster of '0' is ∞] > 0 iff

$\mathbb{P}_p$ [There is ∞ cluster] $= 1$

And $\mathbb{P}_p$ ['0' $\in \infty$ cluster] $= 0$ iff $\mathbb{P}_p$[∞ cluster exists] $= 0$

- We will use 0-1 Law without proof:

It says: An event that depends only on infinitely far out variables (∞-tail) must have prob. 0 or 1

Hence; for any p (parameter) $\mathbb{P}_p$ [exists ∞ cluster] $= 0$ / 1

Let ① $\mathbb{P}_p$ [$C(0)$ is ∞] > 0 then

$\{ C(0) \text{ is } \infty \} \subseteq \{\text{There exists } \infty \text{ cluster}\}$

Hence; $0 \leq P_p\{C(0) \text{ is } \infty\} \leq \mathbb{P}_p$[exists an ∞ cluster]

Idea of pf. $\theta(p) := P_p\,(C(0) \text{ is } \infty)$.

Fix box of side Λ containing 0.

A_Λ is event that $\exists$ ∞-cluster that intersect Λ

$\{C(0) \text{ is } \infty\} \subseteq A_\Lambda$.

$\therefore P_p(A_\Lambda) \geq P_p(C(0) \text{ is } \infty) = \theta(p) > 0$

$A = \bigcup_\Lambda A_\Lambda$ so; $P_p(A) \geq \theta(p) \geq 0$ But By 0-1 $P_p(A) = 1$

If $\theta(p) = 0$

For any finite box Λ.

$E[\#\{x \in \Lambda : C(x) \text{ is infinite}\}]$

$$= \sum_{x \in \Lambda} P_p\,(C(x) \text{ is } \infty) = |\Lambda| \cdot \theta(p) = 0$$

as system has translational sym.

And converse is trivial

<u>origin is connected to ∞</u> $\Rightarrow$ "$0 \leftrightarrow \infty$"

From origin we can find a path (clusters + linking) That intersects box of size Λ for all values of Λ..

<u>Thm</u>: $\exists p_c \in (0,1)$ s.t. for $p < p_c$; $P_p[0 \leftrightarrow \infty] = 0$

& for $p > p_c$; $P_p[0 \leftrightarrow \infty] > 0$

<u>Uniform coupling</u>: Assign each vertex value randomly from 0 to 1. connect vertex only if value $> p$ to make graph of p-probability

Hence; if $p_1 > p_2$ and $0 \leftrightarrow \infty$ in $f(p_1)$

Then $0 \leftrightarrow \infty$ in $f(p_2)$

Thus; $P[0 \leftrightarrow \infty \text{ in } f(p_1)] \leq P[0 \leftrightarrow \infty \text{ in } f(p_2)]$

Hence; if $p_1 \not< p_2$ $P_{p_1}[0 \leftrightarrow \infty] \leq P_{p_2}[0 \leftrightarrow \infty]$

so; $P_p[0 \leftrightarrow \infty]$ is increasing fn

So; For $p < p_c$; $\mathbb{P}_p[0 \leftrightarrow \infty] = \theta(p) = 0$

after that it is non-zero.

Question is what are bounds on p_c??

Ans: $p_c \in (0,1)$

<u>Lemma</u> $p_c > 0$

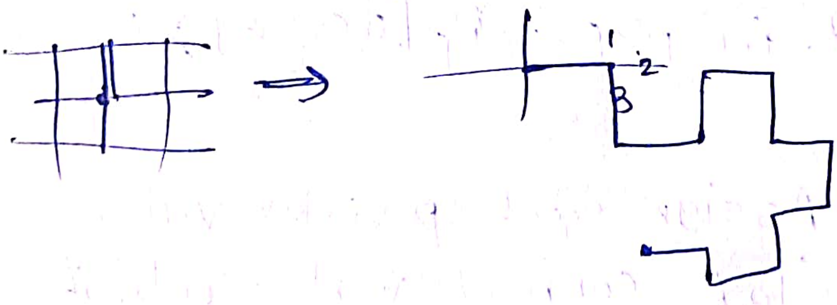
$0 \leftrightarrow \infty \Rightarrow$ for any $l \in \mathbb{N}$; $0 \leftrightarrow l$

Thus; $\mathbb{P}_p[0 \leftrightarrow \infty] \leq \mathbb{P}_p[0 \leftrightarrow l]$

But $\mathbb{P}_p[0 \leftrightarrow l] = p^l \times$ # connected paths of length l.

$= p^l \times 4 \times 3^{l-1}$

But exactly finding such paths would be meticulous. Hence; we find even upper bound on it and ignoring double counting.



→ at each node it has 3 options (indeed there can be less than 3 but we are finding upper bound)

Hence; $\mathbb{P}_p[0 \leftrightarrow \infty] \leq 4p(3p)^{l-1}$

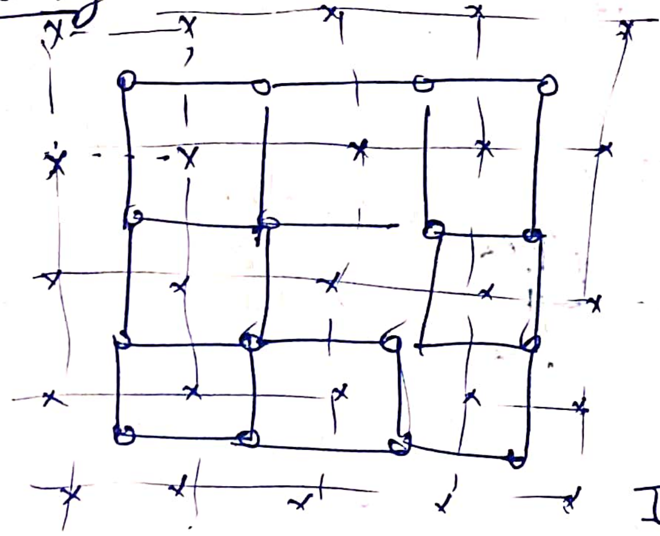
In thermodynamic lt.

$\mathbb{P}_p[0 \leftrightarrow \infty] = \mathbb{P}_p[0 \leftrightarrow l]\Big|_{l \to \infty} \leq 4p \times (3p)^{l-1}\Big|_{l \to \infty}$

So; For $p < 1/3$; RHS vanishes in lt. $\therefore p_c \geq 1/3$

For $P_c < 1$;

Duality



we define a dual ~~g~~ lattice on $\mathbb{Z}^2$

Duality is concept used extensively in CMP, SM, Graph Theory, etc.

It provides interesting insights about original grid.

① Let $\mathcal{L}$ is defined by $\vec{a}_1, \vec{a}_2, \dots \vec{a}_d$; Then dual Lattice $\mathcal{L}^*$ is set of all vectors $\vec{b}$ satisfying: $\vec{b} \cdot \vec{R} = 2\pi k$ where k is any integer. $\vec{R} = \sum_{i=1}^{d} c_i a_i$

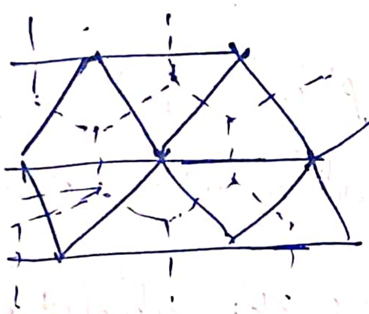
also has basis →

and $\vec{b}$ is called Reciprocal basis vector

s.t. $a_i \cdot b_j = 2\pi \delta_{ij}$ ← used in CMP

② $\mathcal{L}^*$ is grid whose each ~~vertex~~ edge crosses edge of original grid. $\mathcal{L}$. Then $\mathcal{L}^*$ is dual grid.

ex.



Triangular ⟷ Hexa.

Hexa ⟷ Triangular

$\mathbb{Z}^2 \longrightarrow \mathbb{Z}^2$

Note we are restricting to flat grids: $(p-2)(q-2) = 4$

If $(p-2)(q-2) < 4$ platonic solids

$p \to$ ~~No of~~ p-gon

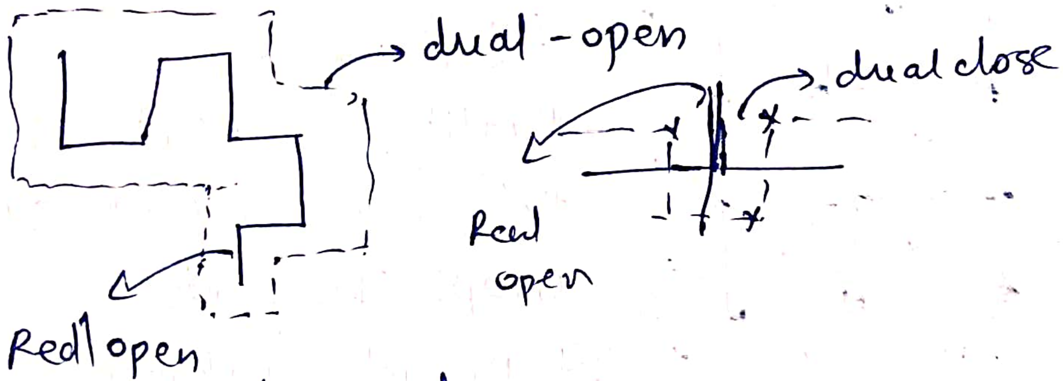
edges concave: Rich percolation

$q \to$ connectivity / No of faces (11)

~~We~~

These is 1-1 correspondene in $\mathcal{L}$ & $\mathcal{L}^*$ of $\mathbb{Z}^2$

Hence; Let's define dual st;



So; dual grid of ~~p~~-grid would give bond percolation of $(1-p)$

Hence if There is an finite cluster in real grid $\mathcal{L}$ containing origin then in $\mathcal{L}^*$ there is cluster which encircles ~~dual grid~~ it.

$O_l :=$ event that length-l dual ckt (in $\mathcal{L}^*$) encircles O

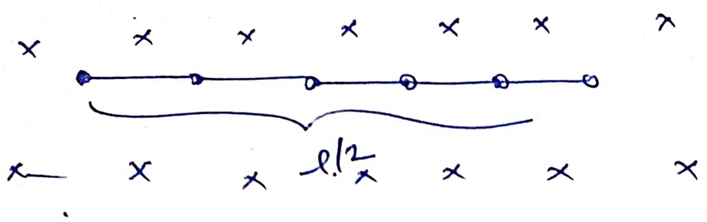
ckt instead of cluster as it is closed.

$$\mathbb{P}_p[O_l] \leqslant \sum_{\substack{\gamma \text{ is ckt} \\ \text{of length } l}} \mathbb{P}_p[\gamma \text{ is all open}]$$

probability that dual vertex $\mathcal{L}^*$'s vertex is open is $(1-p)$ [as $\mathcal{L}^*$ is ~~$\mathcal{L}$~~ bond-percolation with prob. $(1-p)$]

So; $\mathbb{P}_p[O_l] \leqslant (1-p)^l \cdot \#\{\text{length } l \text{ ckt about } 0\}$

Again we found upperbound on it.
we have to make path that loops OG-cluster and has length l.



If there is such a path in $\mathcal{L}$ then it's surrounding path shall have length larger than l.

Hence; Dual ckt shall cross such path befor $l/2$

Thus; we have total $l/2$ places to cut ~~down~~ This path and from such cuttings; length of $l-1$ remains; where it can choose 3^{l-1} ways (ofc with double counting) so;

$$\frac{l}{2} \cdot 3^{l-1} = \frac{l}{6} 3^{l}$$

$$\mathbb{P}_p[O_l] \leq (1-p)^{l} \; \#\{\text{ckt abt } 0 \text{ of length } l\}$$

$$\leq \frac{l}{6} \cdot (3(1-p))^{l}$$

Now; for next step.
we define event: $\mathbb{P}_p[O_{\geq N}]$:= event that dual ckt of length at least N encircles 0.

$$\mathbb{P}_p[O_{\geq N}] \leq \sum_{l=N}^{\infty} \mathbb{P}_p[O_l] \leq \sum_{l=N}^{\infty} l \cdot (3(1-p))^{l}$$

~~This converges for $(1-p) < 1/3$ or $p > 2/3$~~

Intuitively.

Now we have upperbound on prob. that dual ckt of length l encircles origin.

Hence; upperbound ~~existen~~ prob. of existence of such ckt is;

$$\cancel{\sum} \sum_{l=4}^{\infty} \frac{l}{6}(3.(1-p))^{l}$$

$$= \frac{1}{6}\sum_{l=4}^{\infty} \frac{l}{\cancel{6}}(x)^{l} \qquad x = 3(1-p)$$

$$\therefore \quad \sum_{l=4}^{\infty} x^{l} = \frac{x^4}{1-x}$$

$$\frac{\partial}{\partial x}\left(\sum x^{l}\right) = \sum_{l=4}^{\infty} l x^{l} = \frac{x^4(4-3x)}{(1-x)^2}$$

Hence; prob. of such ckt exists $< \dfrac{x^4(4-3x)}{(1-x^2)}$

$$\cancel{\frac{x^4(4-3x)}{(1-x^3)}} = \cancel{\frac{x^4(4-9+3p)}{(1-x^2)}} = \cancel{\frac{x^4(3p-5)}{(1-x^2)}}$$

For $x < 0.766$ This RHS < 1

Hence; for $p > 0.766$ There is non zero probability that no such ckt exists

That is for $p > 0.766$; we 'may' get such ckt

Hence $P_p[0 \leftrightarrow \infty] > 0$ if $p > 0.766$

But. if $P_p(0 \leftrightarrow \infty) > 0$; $P_p[\infty\text{-exists}] = 1$

Here; $p_c < 0.766$ Here upper bound

Critical exponents for 1D percolation

In percolation we have continuous transition between qualitatively different states

Hence critical parameters/exponents

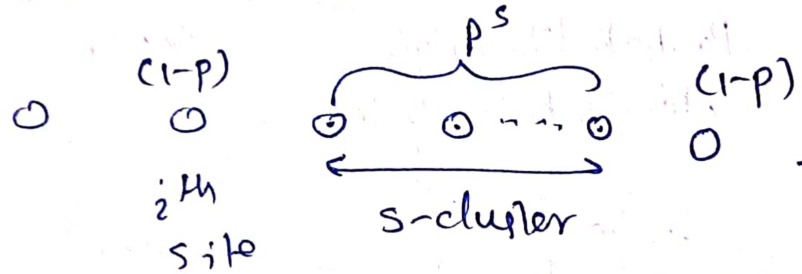
Exponent	Symbol	~~dif.~~ meaning	
- order parameter exponent	β	Probability origin belongs to ∞-cluster	$\theta(p) \sim (p-p_c)^{\beta}$
- Susceptibility	γ	- mean finite cluster size. -How large are clusters before they merge	$\chi \sim \|p_c - p\|^{-\gamma}$
- Correlation length.	ν	Clusters spatial extent.	$\xi \sim \|p-p_c\|^{-\nu}$
- sp. heat	α	cluster generating fn.	$F(p) = \sum_s n_s \cdot s^2$
- cluster size distribution exponent.	τ	How cluster sizes power law at crit.	$n_s(p_c) \sim s^{-\tau}$

We have seen that; $p_c = 1$ for 1D. $\longrightarrow$ we're in Site percolation

① ~~Prob. that~~ site belongs to s-cluster

Q.1 How many s-clusters do we have in chain if length is L, $L \to \infty$.

- each site has prob. $p^s(1-p)^2$ of being Left hand of such cluster ie



$\longrightarrow$ prob is $p^s(1-p)^2$

- Thus total no. of s clusters are $Lp^s(1-p)^2$ Hence;

~~#~~ clusters / site $= p^s(1-p)^2 = n_s$

Q.2 No. of s-clusters / lattice site.

$n_s = p^s(1-p)^2$

Q.3 If we randomly choose a site ~~(occupied)~~ what is probability that it belongs to s-cluster.

it is "$s \cdot n_s$" as there are s-sites in s-cluster.

Hence; since; prob. 'p' is prob. is site is occupied and if it is occupied; then it must belong to cluster. (for now size 1 is cluster too)

$$\therefore p = \sum_s n_s \cdot s.$$

we can show this

$$\sum_s p^s (1-p)^2 s = (1-p)^2 \sum_s p \cdot \frac{d(p^s)}{dp} = (1-p)^2 \cdot p \frac{d(\sum p^s)}{dp}$$

$$= (1-p)^2 p \cdot \frac{d(p/1-p)}{dp}$$

$$= p.$$

Also; $s \to \infty$; $n_s \to 0$ (it is less probable that given site would belog to s-cluster for large s if site is randomly chosen)

~~Q. What is probabiti~~

Q. what is average cluster size that we get if we randomly point at a site on lattice site which is part of finite cluster.

Def: $w_s = \frac{n_s \cdot s}{\sum_s n_s \cdot s}$ = prob. that cluster to which the randomly chosen site belongs to cluster which has exactly s sites.

Thus; average cluster size; S is

$$S = \sum_s s w_s = \frac{\sum_s n_s \cdot s^2}{\sum n_s \cdot s}$$

S also called 'mean cluster size'

$$S = \frac{\sum n_s . s^2}{p} \quad \text{so;} \quad \sum n_s . s^2 = (1-p)^2 \sum_s s^2 p^s$$

$$= (1-p)^2 \left(p . \frac{d}{dp}\right)^2 \left(\sum p^s\right)$$

$$\sum s^2 p^s \Rightarrow p . \frac{\partial}{\partial p}\left(\sum p^s\right) = s \sum p^s$$

$$p . \frac{\partial}{\partial p}\left(s \sum p^s\right) = s^2 \sum p^s$$

$$\sum s^2 p^s = p . \frac{\partial}{\partial p}\left(p . \frac{\partial}{\partial p}\left(\sum p^s\right)\right)$$

~~Hence: $p \frac{\partial}{\partial s}\left(\frac{p}{(1-p)^2}\right) = p\, [$~~

$\cancel{\sum p^s s}$

$$\text{so;} \quad \cancel{p . \frac{\partial}{\partial p}\left(\frac{p}{(1-p)^2}\right)} = p\left[\frac{(1-p)^2 + 2p(1-p)}{(1-p)^4}\right]$$

$$= \frac{p(1-p)^2 + 2p(1-p)}{(1-p)^4}$$

$$= \frac{p(1-p) + 2p}{(1-p)^3} = \frac{3p - p^2}{(1-p)^3}$$

$$\frac{\partial}{\partial p}\left(\frac{p}{1-p}\right) = \frac{(1-p) - p(-1)}{(1-p)^2} = \frac{1}{(1-p)^2}$$

$$\frac{\partial}{\partial p}\left[\frac{p}{(1-p)^2}\right] = \frac{(1-p)^2 . 1 + p . 2(1-p)}{(1-p)^4}$$

$$= \frac{(1-p)^2 + 2p(1-p)}{(1-p)^4}$$

$$= \frac{1-p+2p}{(1-p)^3} = \frac{1+p}{(1-p)^3}$$

$$\text{so;} \quad \sum n_s . s^2 = (1-p)^2 . p . \frac{(1+p)}{(1-p)^3} = \frac{p . (1+p)}{(1-p)}$$

Hence; $S = \left(\frac{1+p}{1-p}\right)$

Q. ~~How does~~ what is probability that two sites separated by r belongs to same cluster.

Ans; $g(r) = p^r$ $g(r)$ is same probability

$g(r)$ is called connectivity fn.

$$g(r) = \exp[\ln(p^r)]$$
$$= \exp[r \ln p]$$
$$= \exp\left[\frac{-r}{(-1/\ln p)}\right]$$

we call $\xi = \frac{-1}{\ln p}$ ← corelation length.

$$\therefore g(r) = \exp[-r/\xi]$$

That is ξ catches essence of how corelation of two sites decays with increasing r ~~at~~ for given p.

for small p; $\xi \to 0$ That is no corelation.

at p close to 1; $\xi = \frac{1}{(p_c - p)}$ $(p_c = 1)$

which is huge. Hence; corelation fn value close to zero; sites become corelated

Also; $S \propto \xi$

and $\sum_{r} g(r) = S$

Q. What is prob. that origin belogs to ~~a~~ cluster

$$\theta(p) = \begin{cases} 0 & ; p < 1 \\ 1 & ; p = 1 \end{cases}$$

Now; calculate $\beta, \gamma, \delta, \nu, \tau$.

β: $\theta \sim (p - p_c)^{\beta}$ $\quad$: $\beta = 0$ Discontinuous

γ: $S = \dfrac{1+p}{1-p}$ $\quad$ as $p \to 1$; $S \to \dfrac{2}{1-p} \sim |1-p|^{-1}$

$\therefore \gamma = 1$

ξ ν: $\xi \sim \dfrac{1}{|1-p|}$ $\quad$ $\nu = 1$

τ: $n_s \sim s^{-\tau}$ $\quad$ $n_s = p^s (1-p)^2$

$= (1-p)^2 e^{s \ln p}$

as $p \to 1$; $\ln p = p - 1$

so; $n_s = (1-p)^2 e^{-s(1-p)}$

~~De~~ as near p_c; $\xi = \dfrac{1}{1-p}$;

$n_s = \xi^{-2} \exp\left(-\dfrac{s}{\xi}\right)$ $\quad$ $n_s \sim s^{-2}$ $\quad$ $\tau = 2$

But why did we do this much hassle?
What's spl bout critical exponents?

1. critical parameters are seen to be divergent at critical point.
2. Near critical point, their behaviour is explained by power-law
3. Later we will see different systems may have same critical exponents
4. ~~It follows~~ Scaling follows formula.

$$\boxed{2 = \alpha + \gamma + 2\beta}$$

$\gamma = 3-\tau/8$

$\beta = \tau - 2/8$

$2-\alpha = \tau - 1/8$

↑ Regardless of what lattice you are on.

5. for d-dim, we get:

$$\boxed{d\nu = \gamma + 2\beta = 2-\alpha}$$ Hyperscaling law ($d_{upper} = 6$)

6. Interestingly: These laws are phenomenological
for $d \geq d_{upper}$ exponents same as ∞ dim bethe.

$M_0 \propto |p-p_c|^{2-\alpha}$

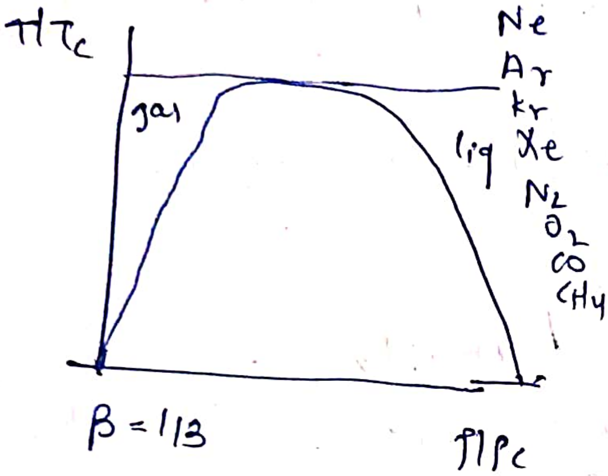
<u>Other models</u>

	α	β	γ	δ	ν	η
3d-ising:	0.10	0.33	1.24	4.8	0.63	0.04
water-gas	-0.014	0.326	1.24		0.63	

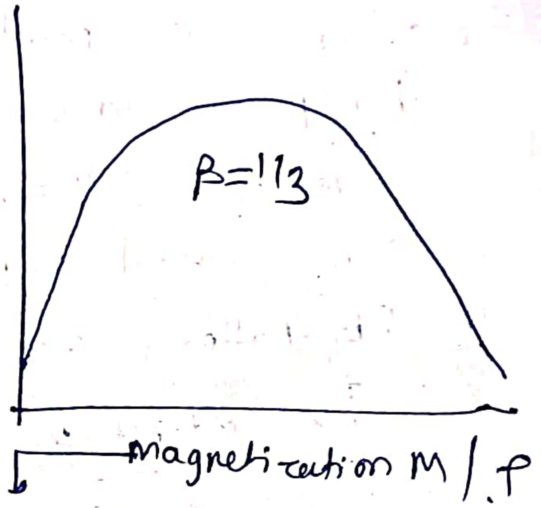
~~Q. is How do we get~~

We ~~now~~ phenomenologically observed that some systems have same critical exponents even though microscopic constituents are different.

- ex. T/T_c vs ρ/ρ_c.



- ex.



Uniaxial ~~ferro~~ antiferro MnF_2

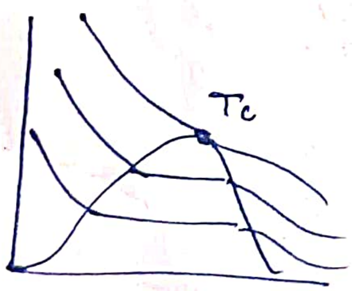
- Phase sep^n in binary fluid $CCl_4 + C_7HF_{16}$

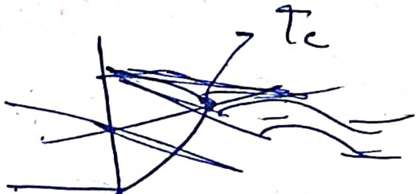
Ref $\longrightarrow$ Phase transition in statmech

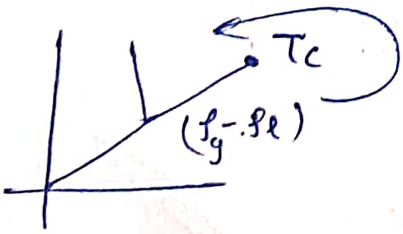
So; what is this??

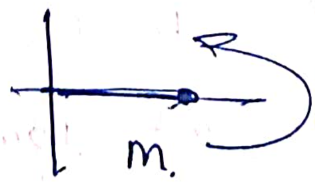
How can two seemingly uncorrelated system act same near critical pt.

Ex. Lattice gas & ising.









This prompted scientists to ~~define~~ say universality:
If two systems have same critical exponents they belong to same universality class